Contractive Orbits of Set-Valued Maps and Applications to Fixed Points

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Abstract

We study a class of set-valued maps that contract along an orbit to a point that is either a strict fixed point or lies outside the domain of the map. Such mappings arise naturally in variational analysis and optimization. An extensive study of mappings with such properties in metric and generalized metric spaces can be found in [1].

In [2], we extend the notion of a contractive orbit with respect to the topology of the underlying space. We employ the technique developed by Kupka in [3] to prove the existence of a strict fixed point or a point outside the domain of the set-valued map. As an application, we derive a sufficient condition ensuring that a function attains a strong minimum in generalized topological settings. We also define a contractive orbit with respect to a generalized distance function and investigate conditions under which these two notions of a contractive orbit coincide. Finally, we present examples of commonly used distance functions that are not true metrics but nevertheless satisfy our assumptions.

[1] M. Ivanov, D. Kamburova, N. Zlateva, *On Long Orbit Empty Value (LOEV) principle*, Optimization, to appear.

[2] D. Kamburova, *Hausdorff characterizations of first countable T1 spaces via fixed point theorems*, https://arxiv.org/pdf/2601.17380v2;

[3] Kupka, I., *A Banach fixed point theorem for topological spaces*, Revista Colombiana de Matematicas, XXVI, 95–100, (1992).